CS 692 Capstone Exam Theory Spring 2024

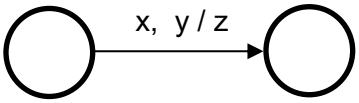
Choose any 2 of the 3 problems. If all three are answered, only questions 1 and 2 will be graded.

1. Consider languages A = { $a^m b^m c^n$ | m, n ≥ 0 } and B = { $a^m b^n c^n$ | m, n ≥ 0 } over Σ = {a, b, c}:

   (a). (8 points) Give a context-free grammar (CFG) for language A.

   (b). (12 points) Draw a push-down automaton (PDA) for language B.

Please use the following notation to label the transitions in the PDA:



(read input symbol x, stack top is y, push symbol z)

2. Consider language C = { $a^n b^n c^n$ | n ≥ 0 } over Σ = {a, b, c}:

   (a). (8 points) Give the Pumping Lemma for context-free languages.

   (b). (12 points) Prove that language C is not context-free using Pumping Lemma.

3. (20 points) Prove that the "halting problem" language HALT = { <M, w> | the Turing machine M halts on input w } is undecidable. You may assume that the following language A_{TM} = { <M, w> | M is a Turing machine that accepts w } is known to be undecidable.